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Dynamical Systems & Nonlinear Partial Differential Equations

FINAL REPORT

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13. ABSTRACT *(Maximum 200 words)*

This final report deals primarily with the study of infinite dimensional dynamical systems and, more specifically, with hyperbolic and parabolic partial differential equations. Part of the final report is devoted to showing that a nice dynamical system is defined for specific types of equations that occur often in models for physical systems. The remainder of the report belongs to the general category of the investigation of the qualitative properties of infinite dimensional flows; in particular, asymptotic behavior of solutions, stability and instability, scattering theory, attractors and Morse decompositions.

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WORK BY C.M. DAFERMOS

A subject of investigation was the extent to which an entropy inequality (i.e., the Second Law of Thermodynamics) induces stabilization of solutions of hyperbolic systems of conservation laws. In [1] it was shown that the entropy inequality guarantees uniqueness of Lipschitz solutions within the class of BV solutions provided that the entropy is convex just in certain directions compatible with the natural invariance of the system expressed in terms of "involutions". In [7] it was proved that BV solutions of strictly hyperbolic systems with shocks of moderate strength, which satisfy the Liu admissibility condition, minimize the rate of total entropy production.

The theory of generalized characteristics for a single conservation law, developed earlier by the author, was applied in [3] and [5] to conservation laws with inhomogeneity and fading memory. In [8], the theory of generalized characteristics was developed for systems of conservation laws and was used to obtain information on the large time behavior of solutions.

References

[1] "Quasilinear Hyperbolic Systems with Involutions", *Arch. Rational Mech. Analysis*, 94 (1986), 373-389.

[2] "Estimates for Conservation Laws with Little Viscosity", *SIAM J. Math. Analysis*, 18 (1987), 409-421.

[3] "Trend to Steady State in a Conservation Law with Spatial Inhomogeneity", *Quart. Appl. Math.*, XLV (1987), 313-319.

[4] "Solutions in L^∞ for a Conservation Law with Memory", *Analyse Mathematique et Applications*, Gauthier-Villars, Paris (1988), 117-128.

[5] "Solutions with Shocks for Conservation Laws with Memory", *Amorphous Polymers and Non-Newtonian Fluids* (C. Dafermos, J.L. Ericksen, and D. Kinderlehrer, Eds.) Springer-Verlag, New York (1987), 33-55.

[6] "Hyperbolic conservation laws with memory", *Differential Equations* (C. Dafermos, G. Ladas and G. Papanicolaou, Eds.) Marcel Dekker, New York, (1989), 157-165.

[7] "Admissible wave fans in nonlinear hyperbolic systems", *Arch. Rational Mech. Analysis*, 106 (1989), 243-260.

[8] "Generalized characteristics in hyperbolic systems of conservation laws", *Arch. Rational Mech. Analysis*, 107 (1989), 127-155.

WORK BY J. HALE

The work of Hale was directed toward understanding the structure, dynamics, and bifurcations of the attractors in dissipative infinite dimensional dynamical systems. One such class of systems studied was a damped nonlinear hyperbolic (wave) equation; of particular interest was the limit of the attractor to that of a parabolic equation as the damping became large; see [1], [6]; also [8], [9], [11]. Another class of systems studied was nonlinear parabolic equations with small diffusion. In [7], [10] the genesis an time-evolution of transition layers for such systems was investigated. Finally, chaotic phenomena within the attractors of differential delay equations was studied [2], [5].

References

[1] Limiting Behavior of a Linearly Damped Hyperbolic Equations. 86-6.

[2] Homoclinic orbits and Chaos in Delay Equations. 86-

[3] Compact Attractors for Weak Dynamical Systems. 86-22.

[4] Some Examples of Infinite Dimensional Systems. 87-

[5] Onset of Chaos in Differential Delay Equations. 87-2.

[6] Upper Semicontinuity of the Attractor for a Singularly Perturbed Hyperbolic Equation. 86-40.

[7] Existence and Stability of Transition Layers. 87-27.

[8] Shadow Systems and Attractors in Reaction-Diffusion Equations. 87-28.

[9] Lower Semicontinuity of Attractors of Gradient Systems and Applications. 87-38.

[10] Slow Motion Manifolds Dormant Instability and Singular Perturbations. 88-9.

[11] Lower Semi-Continuity of the Attractor for a Singularity Perturbed Hyperbolic Equation. 88-19.

WORK BY J. MALLET-PARET

Mallet-Paret's work focussed on the dynamics of high and infinite dimensional dissipative dynamical systems. Several broad classes of systems which exhibit particularly simple dynamics were identified in [5], [6] and [8]. These are systems with enough monotonicity that a Poincaré -Bendixson theorem holds, and include autonomous scalar reaction-diffusion equations. Even in the absence of monotonicity some generalizations of these systems, for example differential-delay equations with negative feedback, exhibit a Morse structure in the attractor ([1], [3], [4]). Studies of such systems were also done from the point of view of singular perturbations ([3], [7]). Finally, in [8] we obtained an existence result for an inertial manifold for a higher dimensional reaction diffusion equation. This result was notable in that the usual spectral-gap condition did not hold; in its place, a more general condition (the Principle of Spatial Averaging) involving both eigenfunctions and eigenvalues was used.

References

[1] Morse decompositions for delay-differential equations, J. Differential Equations, 72 (1988) 270-315.

[2] Inertial manifolds for reaction-diffusion equations in higher space dimensions (with G.R. Sell), Jour. Amer. Math. Soc. 1 (1988) 805-866.

[3] Transition layers for singularly perturbed delay differential equations with monotone nonlinearities (with S.N. Chow and X.B. Lin), Jour. Dyn. Diff. Eq. 1 (1989), 3-43.

[4] Connections between Morse sets for delay-differential equations (with B. Fiedler), Jour. Reine Ang. Mat. 397 (1989), 23-41.

[5] The Poincare-Bendixson theorem for scalar reaction diffusion equations (with B. Fiedler), Arch. Rat. Mech. Anal., 107 (1989), 325-345.

[6] The Poincare-Bendixson theorem for monotone cyclic feedback systems (with H.L. Smith), J. Dyn. Diff. Eq., 2 (1990), 367-421.

[7] A differential-delay equation arising in optics and physiology (with R.D. Nussbaum), SIAM Jour. Math. Anal. 20 (1989), 249-292.

[8] Stable periodic solutions for the hypercycle system (with J. Hofbauer and H. L. Smith), J. Dyn. Diff. Eq., 3 (1991), 423-436.

WORK BY P.E. SOUGANIDIS

Souganidis continued his work on the weak formulation of propagating fronts and/or its relations to applications like phase transitions, flame propagation etc. Among the completed projects are:

(i) Phase transitions and generalized motion by mean curvature (with L.C. Evans and H.M. Soner).

(ii) Front propagation and phase field theory (with G. Barles and H.M. Soner).

(iii) Uniqueness of rotationally symmetric surfaces moving by mean curvature (with H.M. Soner).

Souganidis and P.-L. Lions have established the convergence of second order accurate TVD schemes for scalar conservation laws and one dimensional Hamilton-Jacobi equations.

References

[1] Stability and Instability Results for KdV Type Waves. 86-38.

[2] A Uniqueness Result for Viscosity Solutions of Second-Order Fully Nonlinear Partial Differential Equations. 87-12.

[3] A PDE Approach to Certain Large Deviation Problems for Systems of Parabolic Equations. 87-22.

[4] A PDE Approach to Geometric Optics for Certain Reaction-Diffusion Equations I. 87-23.

[5] Viscosity Solutions of Second-Order Equations, Stochastic Control and Stochastic Differential Games. 87-34.

[6] Wavefront Propagation for Reaction-Diffusion Systems of PDE. 89-3.

WORK BY WALTER STRAUSS

R. Glassey and W. Strauss have made progress on several fronts in the kinetic theory of plasmas. (i) For the relativistic Vlasov-Maxwell equations (RVM), they prove [1] the absence of singularities for initial data without compact support. (ii) They have a similar result in the presence of a Fokker-Planck collision term. (iii) Although the Jacobian of the collision operator $u, v \to u', v'$ for relativistic particles at a fixed scattering angle is unbounded, they show that its average over all the angles is bounded.

M. Grillakis, J. Shatah and W. Strauss have completed their work on stability of solitary waves with a Lie group of symmetries. They have found that rotations tend to stabilize such systems of wave equations.

P. Souganidis and W. Strauss have determined the conditions for stability and instability for the solitary waves of the BBM equation. They showed, for the first time, that solitary waves of the KdV type can be unstable.

W. Craig, T. Kappeler and W. Strauss have proved the infinite gain of regularity for a quite general class of equations. This generalizes the work of T. Kato on the KdV equation.

Strauss has completed a CBMS monograph based on his lectures at the CBMS-NSF conference.

References

[1] (1986) Singularity formation in a collisionless plasma could occur only at high velocities, Arch. Rat. Mech. Anal. 92, 59-90 (with R. Glassey).

[2] (1986) Time-periodic scattering of symmetric hyperbolic systems J. Math. Anal. Applics. 122, 444-452 (with J. Cooper).

[3] (1986) On the scattering frequencies of time-dependent potentials Math. Meth. in Appl. Sci. 8, 576-584 (with J. Cooper and G. Perla Menzala).

[4] (1986) Instability with constraints, in: Semigroups, Theory and Applications, Res. Notes in Math., Pitman Publ., 1986, p.227-234 (with J. Shatah).

[5] (1986) On the stability under dilations of nonlinear spinor fields, Phys. Rev. D 34, 641-643 (with L. Vázquez).

[6] (1987) On the dynamics of a collisionless plasma, in: Dynamical Problems in Continuum Physics, I.M.A. Volume 4, Springer Verlag, 1986, p. 186-192 (with R. Glassey).

[7] (1987) Imaginary poles of radial potentials, Mat. Aplic. Comput. 7, 17-24.

[8] (1987) High velocity particles in a collisionless plasma, Math. Meth. in Appl. Sci. 9, 46-52 (with R. Glassey).

[9] (1987) Stability theory of solitary waves in the presence of symmetry I, J. Funct, Anal. 74, 160-197 (with M. Grillakis and J. Shatah).

[10] (1987) Stability and instability of solitary waves of KdV type, Proc. Royal Soc. London, A411, 395-412 (with J. Bona and P. Souganidis).

[11] (1987) Absence of shocks in an initially dilute collisionless plasma, Comm. Math. Phys. 113, 191-208 (with R. Glassey).

[12] (1988) Numerical computation of the scattering frequencies for acoustic wave equations, J. Comput. Phys. 75, 345-358 (with M. Wei and G. Majda).

[13] (1989) Computation of exponentials in transient data, IEEE Trans. on Antennas and Propagation 37, 1284-90 (with M. Wei and G. Majda).

[14] (1989) Stability theory of solitary waves in the presence of symmetry II, J. Funct. Anal. (with M. Grillakis and J. Shatah), accepted.

[15] (1989) Nonlinear Wave Equations, NSF-CBMS Research Monograph, Amer. Math. Soc., accepted.

[16] (1989) Large velocities in the relativistic Vlasov-Maxwell equations, J. Fac. Sci. U. of Tokyo, IA, 36, No. 3 (with R. Glassey), accepted.

[17] (1989) Instability of a class of dispersive solitary waves, Proc. Royal Soc. Edinburgh (with P. Souganidis), accepted.

[18] (1989) Infinite gain of regularity for dispersive evolution equations, in: Microlocal Analysis and Nonlinear Waves, IMA (with W. Craig and T. Kappeler), accepted.